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MONEY, CYCLES AND COMPLEXITY

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Abstract

The main idea of the present paper is that the appropriate inclusion of money and monetary circulation into economic analysis implies shift from acyclic to cyclic economic mathematical models. In the same tame such transition allows for the analysis of more complex economic instances, complexity being viewed in terms of computational complexity. Remarkably, the emergence and development of complex cyclical economic systems is actually possible only in the context of risk and uncertainty.

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1. Introduction

Intuitively, the monetary circulation and the complexity of economic systems should be strongly connected. The money allows for the elimination of the double coincidence of wants (Jevons, 1875) and broadens the scope of exchange between economic agents. However, the elimination of the exchange gridlock on bilateral level via the introduction of money, simply conveys the problem to the seller who is taking the relay (see for discussion Marx, 1887). Consequently, the exchange process must be infinite since otherwise the last seller will remain unsatisfied. Such infinite process can take only the form of circulation of money, that is of closed cycles of motion of money, since otherwise there will be accumulation of stocks of money in some agents and stocks of goods in others. On macro level this is the concept of circular flow of income (money), present since the works of W. Petty (Petty, 1997 [1899]), Cantillon (see Murphy, 1993), Marx (1887), Keynes (1936) and others.

The paper is structures as follows. The second part deals with the role of money in constructing consistent price systems. The main idea is that we can define money only against all the other goods and that the price level is nothing but the relative price of money. This means that money is neither neutral nor homogenous. Technically this conclusion is based on the assumption that the bilateral price matrix is cyclic, what is against neoclassical assumptions and have important consequences in terms of complexity of economic systems.

The third part is dedicated to the analysis of circulation of money and money velocity in particular. The main conclusion is that we can, contrary to the neoclassical theory, derive micro behaviour from such macro parameters as income velocity and price level (inflation). This deduction means that macroeconomic policy may affect individual agents behaviour and complexity configuration of the economy.

The fourth part discusses directly the complexity of the economic systems from the point of view of computational complexity the markets are assumed to resolve. The neoclassical paradigm is inherently based on inconsistently introduced acyclic structures, which are relatively simple. The appropriate incorporation of money and financial institutions in economic system formulation, implies admittance of more complex structures, based on Eulerian and Hamiltonian cycles. Paradoxically, complex economic constructions can emerge and develop only in environments, involving risk and uncertainty.

2. Relative Prices and Price Level: The Complex Nature of the Numéraire Function of Money

The central purpose of the orthodox economic theory and the neoclassical school in particular, is to prove that decentralized economic exchange can guarantee equilibrium convergence and optimal allocation of resources. Since general equilibrium is hardly possible without some kind of individual goods mutual comparability, the neoclassicals focused their attention on such metaphysical monetary attributes, as homogeneity postulate (see for example Takayama, 1990) and money neutrality (Lucas, 1995). Both homogeneity and neutrality are quite suitable in order to pledge commensurability among goods, without any tangible impact on exchange and production. This simplification ignores the point, that equally in physics and economics, it is not possible to measure an object, without affecting it.

Following these two basic neoclassical assumptions, the price of money is artificially fixed, before the existence of equilibrium is proved. This can be done in two ways. The first variant is simply to fix the price of money to unity (Walras, 1874), while the second implies the fixing of the price level. Both variants are equivalent. Walras explicitly states that we should measure the monetary prices in terms of monetary units and not in terms of the exchange value of the monetary unit. This is certainly true in the case of hypothetical simultaneous barter exchange at equilibrium prices. However, if we swap goods for money and vice versa and the exchange system is not in equilibrium, we are interested in knowing not only the monetary prices of the specific goods and services, but the exchange value of money itself, since the latter, according to Pigou (1917), is nothing but an expression of the inverse of the sum of the prices of all exchanged goods in terms of money, and consequently affects the outcome of the exchange. This means that in disequilibrium situations we should abandon the Walrasian postulate about fixing the exchange value of money. The convergence to equilibrium is thus simultaneous adjustment of all prices, including that of money. To put it differently, the price of money and all the other prices are interdependent; we cannot fix one of the prices in advance. The particularity of the exchange value of money is that it consists of combination of all the outstanding prices. Therefore, the price of money combines micro and macro features.

If all markets, including that of money, are in equilibrium, then homogeneity hypothesis holds in the sense that if we multiply all prices by factor k ($\infty > k > 0$) supply and demand functions remain unaltered. However, even in such a case, if all prices include the price (exchange value) of money, then it is simply impossible to multiply every single price by the same factor (the price of money moves in opposite direction to all other prices). Consequently the more sophisticated adepts of the neoclassical school acknowledge that homogeneity can be applied to all goods, except money (Modigliani, 1944), but several authors (Lange, 1942; Patinkin, 1947; 1949) object that if homogeneity is applied to all but one price, then according to Walras Law, equilibrium on non-money markets, combined with the Cambridge money demand equation, would imply that money market is also homogenous of degree zero.

There is another logical conundrum. In order to preserve the neutrality and homogeneity, the neoclassical theorists (with the somewhat paradoxical exception of Walras himself), separate the processes of deriving relative prices and the price level. This is necessary in order to prove the existence of equilibrium without money and next to prove that money market is also in equilibrium. According to Fisher (1963), the procedure includes two steps. First, the real sector determines equilibrium relative prices and quantities, exchanged at equilibrium, usually via some variant of tâtonnement process. Then money market is introduced and the equilibrium price level is derived. In other words, the relative prices are established without any relation to currency.

This is nonetheless false. If we have only two goods, the relative price can be easily determined, since one of the goods can be viewed as money. But if we have more than two goods, the relative prices can be expressed only in terms of bilateral exchange proportions, forming a system, analogous to the system of bilateral exchange rates.

Such systems obey consistency rule excluding arbitrage profits and any good's bilateral exchange proportions against all the other goods contain all information about the entire number of cross exchange proportions, as Walras himself once demonstrated. This means that in such a system any good, exchangeable directly or indirectly against all the other goods, may be viewed as numéraire. In mathematical terms this means that the matrix of relative prices must be indecomposable (what is the usual condition of input-output and general equilibrium analysis) and imprimitive (cyclic), what is against the standard general equilibrium features. This, as we shall see later, leads us to complexities, that strongly affects the behaviour of money intermediated economic systems.

We start the more detailed exposition by introducing the square n-matrix of relative prices A. The elements a_{ij} of this matrix are defined in a similar manner as in the case of input-output analysis, i.e. they represent the quantity of good i exchanged for unit of good j, or elements a_{ij} stand for the price of j in terms of i. All elements are non-negative, or $a_{ij} \geq 0$. More precisely the elements a_{ij} are equal to zero when there is no exchange between i and j and are positive in the opposite case. When exchanging i for unit of good j a sum of money a_{ji} is paid. When $a_{ij} \neq 0$, we have $a_{ij} = {}^{1}/_{a_{ji}}$, where a_{ji} is the price of money (good i) in terms of j.

Proposition: The necessary condition for the existence of price level in terms of all exchanged goods is the matrix A to be indecomposable and the sufficient requirement is the above matrix to be in addition imprimitive.

Proof of necessity:

We start with the definition of indecomposable matrix using the notation of Takayma (1990). Matrix A is called decomposable if there exists partition $\{J, K\}$ of $N = \{1,2,3, \dots, n\}$, such that $N = J \cup K$ and $J \cap K = \emptyset$ with $J \neq \emptyset$ and $K \neq \emptyset$ and $a_{ij} = 0$ for $i \in K$ and $j \in J$. The equality $a_{ij} = 0$ means that there is no exchange between goods i and j. This decomposition is interpreted in the sense that the market is partitioned into two groups, the J group and K group. Goods that belong to J group are not exchanged for goods belonging to K group. From the definition of decomposability if follows that if the matrix A is indecomposable, then the above partition does not exist so $J \cap K \neq \emptyset$ and there are no groups of goods that are not connected via exchange. This condition is obviously necessary in order to be able to express all prices in terms of one good, representing the price level.

The proof of sufficiency is more complicated and is related to imprimitive or cyclic square matrices. An $n \times n$ indecomposable matrix A is called imprimitive if:

(i) There exists a partition $\{J_1, J_2, \dots, J_m\}$ of $N \equiv \{1,2, \dots, n\}$ such that $N = J_1 \cup J_2 \cup \dots \cup J_m, J_i \cap J_j = \emptyset \ (i \neq j), J_i = \emptyset, i = 1,2, \dots, m, and$

(ii) $a_{ij} = 0(i \notin J_{i-1}, j \in J_i), and \ \sum_{i \in J_{i-1}} a_{ij} > 0(j \in J_i), i = 1,2, \dots m.$

We assume that J_0 is the same as J_m. In the case of square matrix $m = n$.

The fact that A is imprimitive means that any market which belongs to the J_i-group of goods does sell its goods against the J_{i-1}-group of markets but does not unload its goods to any other group of markets. We assume also that symmetrically the J_{i-1}-group pays respective sums of money only to J_i-group. If every market is viewed as separate group, then we can order markets in succession such that good $i-1$ is exchanged for good i and the m-th good is exchanged against the first good thus closing the cycle. The same procedure can be applies for the cycles implying payments from i-th against $i-1$-th market.

Any $i-1$-th good in the first cycle plays the role of numéraire in respect to i. The matrix may have several cyclical decompositions. If such succession is feasible, then we can express any bilateral price via the prices of all the other bilateral prices, or $a_{i-1,i} = a_{i,i+1} \dots a_{n-1,n} a_{n,1} a_{1,2} \dots a_{i-2,i-1} a_{i,i-1}$. The equality holds because we assume lack of arbitrage profits. Since $a_{i,i-1} = {}^1\!/_{a_{i-1,i}}$, we have $\left(a_{i-1,i}\right)^2 = a_{i,i+1} \dots a_{n-1,n} a_{n,1} a_{1,2} \dots a_{i-2,i-1}$ or $a_{i-1,i} = \sqrt{a_{i,i+1} \dots a_{n-1,n} a_{n,1} a_{1,2} \dots a_{i-2,i-1}}$. The price of the good, fulfilling the role of numeraire in the bilateral exchange is $a_{i,i-1} = {}^1\!\Big/_{\sqrt{a_{i,i+1} \dots a_{n-1,n} a_{n,1} a_{1,2} \dots a_{i-2,i-1}}}$. We can express any price in terms of money (money is any arbitrary $i-1$th good). In particular, the price on nth good in terms $i-1$th one is $a_{i-1,n} = a_{i,i+1} \dots a_{n-1,n}$. This simply means that having even one relative bilateral price defined, we have also the price level in terms of respective $i-1$th good, since all relative prices are connected via closed chain. This completes the sufficiency.

All this means that we cannot separate the price level definition and the relative prices formation. It follows also that by changing relative prices we affect the price level and vice versa, consequently money is neither neutral nor homogenous. This is so because price level variations affect the relative price of money and the variations of the price of money itself are inversely related to the price level. In addition, we have as many price levels as goods, involved in the exchange.

This indicates also that isolating money as separate good, outside the system of simultaneous equations, is wrong.

Including money in the system of equations however also creates problems, especially if our objective is to prove the existence and the stability of the general equilibrium. The main difficulty is the requirement that price matrix A must be imprimitive. This contradicts the traditional approach to equilibrium. From the matrix theory we know, that a non-negative indecomposable matrix is primitive if it has at least one diagonal element which is positive (Takayama, 1990)- in the case of imprimitive matrices all diagonal elements are zeros.

In the case of price matrix A this condition may be fulfilled if one the good's prices is defined vis-à-vis itself. By subtracting the money market from the system of markets and by fixing illegitimately its price to unity, the neoclassical economists are doing exactly this. It is true, that in the case of commodity money, we can arbitrary fix the quantity of the respective good that is used as numéraire.

For example, the official gold content of the US dollar maybe fixed as $\frac{1}{38}$ troy ounces per dollar or equivalently 38 dollars per troy ounce. But this does not mean that the relative price of the troy ounce in terms of the other goods and the price (the inverse of the purchasing power) of the dollar are fixed, because the exchange proportions between gold (dollar) and the other commodities vary according to supply and demand conditions. Fixing the value of money is the actual reason behind the famous compensated dollar proposal of I. Fisher, i.e. we can have stable price of money only if we vary the gold weight of the unit. Any change of the purchasing power of the dollar modifies its real gold content and vice versa, so that we cannot distinguish between unit and price change if we observe only the purchasing power of money. It is interesting that A. Sen (1977) comes to similar conclusion in the case of cardinal full comparability of personal welfares in the sense that it is impossible to distinguish between welfare and unit of measurement changes.

To support Fishers recommendations we should admit, that the predisposition for persisting alternating declines and increases in prices is a basic characteristic of gold standard (Cagan, 1984). At an earlier time the instability of the exchange value of gold was discussed by Marshall (1886) and Jevons (1875), who supported the so called "tabular standard", a predecessor of the modern monetary targeting. Theoretical simulations, based on DSGE model, including gold sector (see Bordo, Dittmar and Gavin, 2007), generally confirm these observations. Both inflation and price level targeting provide more short-run price stability than does the gold standard.

The self-referential way of fixing the price of money is at the origin of the inconsistency of the neoclassical theory of money. Money can be defined only in respect to all the other goods. Finally, if we fix the price level to unity, the matrix A can be artificially defined as primitive, but we cannot express the relative prices and the price level in a consistent way. Alternatively, if we define money in terms of all the other exchanged goods we obtain a system which is unstable and does not automatically converge to equilibrium (see Ganchev, 2015), but can explain complex real world phenomena such as boom and bust cycles, financial bubbles and instability.

3. The Circulation of Money and Velocity

Monetary economy or economy were the exchange is intermediated by money, can be only economy with production since pure exchange means that after the exchange agents with remaining money stocks are unable to use them in any meaningful way. Only closed cycles of money circulation transforms the latter into useful device. Circulation in turn is impossible but under repeated productive cycles. Circulation and velocity of circulation in particular is key variable reflecting the reproductive parameters of the economy. So the income velocity of money is the way the production is introduced in this paper.

We start with the assumption that the economy is producing below its production possibility frontier. Second, the economic agents are endowed with indirect utility functions, so the utility they derive via exchange depends on income and the relative prices. The question is how the monetary prices of individual goods are "discovered" in the process of exchange. The neoclassical analysis relies on mechanisms such "tâtonnement" which are not well defined. We can suggest an alternative, monetary circulation based mechanism.

Suppose we have n economic agents and m goods (including money), produced and exchanged between agents with the intermediation of money. For a given time period every agent i performs v_{ij} number of payments against agent j (i.e. the volume of payments divided by money endowments of agent i). The square matrix $V = [v_{ij}]$ describes the exchange between agents. The matrix V is nonnegative and indecomposable by definition, since it is assumed that there are no groups of agents that are not connected via exchange to the other participants.

Every economic agent possess some quantity of money m_i so we have $m_1 + m_2 + \dots m_i + \dots m_n = M$. In principle the quantity of money per agent varies with the time since the participants spend money and receive payments.

Further we can write $V \cdot m = \lambda m$, were λ is the eigenvalue of the matrix V and m is the eigenvector of average monetary endowments. Obviously we have also $\lambda m = \lambda m_1 + \lambda m_2 + \dots \lambda m_n = \lambda(m_1 + m_2 + \dots m_n) = \lambda M$. We interpret λ as the (macroeconomic) velocity of money. The meaning of the fact the income velocity of money is an eigenvalue (Frobenius root) of the matrix of the individual velocities is that in such a case, every economic agent can generate income $y_i = m_i \lambda$. In other words, velocity as eigenvalue, presupposes a coordination of bilateral exchanges that allows for the participation of all agents in proportion to their average money holdings. Without such condition money intermediated exchange including all agents and reproducing their money endowments via closed circulation cycles, would not be possible.

Since the money is spent in exchange for goods, the velocity of money, taking into account the price level, determines the physical quantity of goods produced and exchanged between agents and the monetary price per physical unit respectively. So given the velocity and the price level, we have also relative prices. In other words, we introduce indirect utility function of the following type $f_i(P, m, \lambda) = v_i(p, y_i) = u_i(x(p, y_i))$, where P is the price level, m is vector of average quantities of money per economic agent, λ stands for the macroeconomic velocity of money, p is the vector of relative prices and y_i is the income of agent i. Further we can write $f_i(P, m, \lambda) = F_i(P, \lambda)$ since m is an eigenvector associated with λ. The eigenvector m is unique up to a scalar multiple, that's why we need the price level P to determine the real quantity of money endowments. Note also that if the economic agents observe the relative prices, they observe also the price level and vice versa.

Thus the utility of any economic agent depends on the price level and velocity. To close, we obtain an individual utility function, depending on macroeconomic variables, contrary to neoclassical bottom up approach.

The derivation of the income velocity of money as a characteristic root of the matrix of bilateral velocities means, that the velocity of money is an extremely important macroeconomic device, directly dependent on the micro level, but irreducible to it. It means also, that any particular velocity is associated with the respective money endowments (the eigenvector m). In principle, the velocity should be considered unobservable and especially volatile variable. The importance of the conclusion, that macroeconomic parameters determine micro optimizing behaviour is related to the opportunity to use macroeconomic policy beyond rational expectations channel, since policy decisions directly affect individual preferences and have real consequences. Thus macroeconomic policy shapes micro exchange networks, as it will be discussed in the next section.

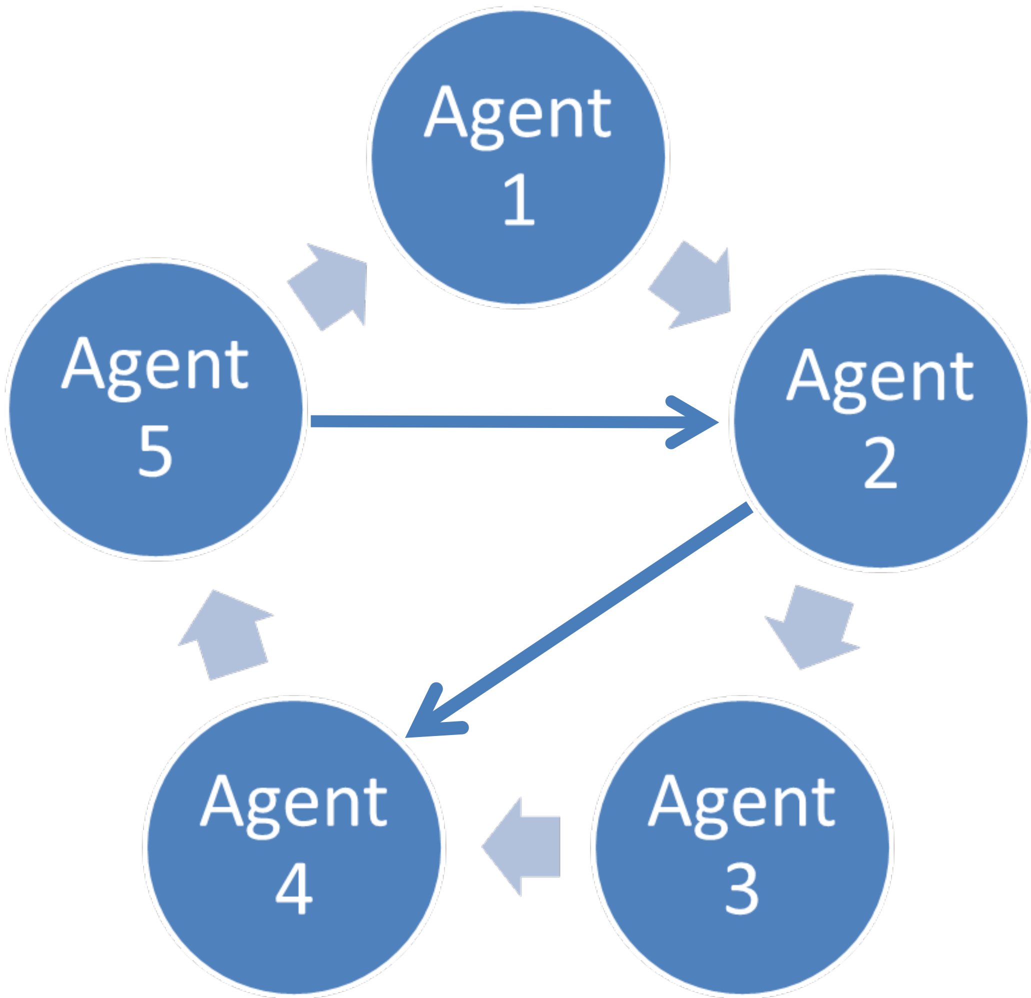
4. Monetary Circulation, Directed Graphs Theory and Complexity

We continue our exposition with the equation of exchange $Mv = Py$ were , v, P and y are as usual the quantity of money, the velocity, the price level and the real income. While the right hand side of the equation of exchange is more or less obvious, the left hand side is not trivial. The problem is the velocity of money, which has different interpretations.

Another interpretation of indecomposable (irreducible) matrices is in terms of directed graphs. We associate with every matrix V a graph G_v with n vertices. The elements v_{ij} correspond to the edges of the graph. The matrix V is irreducible if and only if the graph G_v is strongly connected (Sikolya, 2005). Further, a directed graph is defined as strongly connected if there exists a path in each direction between each pair of vertices of the graph. This means that cycles, including all vertices (economic agents), exist, what corresponds to the circulation of money in a productive economy.

The graphs can be simple and directed or digraphs. In the latter case the vertices are connected by arrows. The Figure 1 shows a directed graph forming a closed loop.

Figure 1



In the Figure 1 the vertices represent the respective agents or sectors of the economy while the edges visualize cash flows between the sectors. The existence of monetary flows forming a closed cycle is a necessary precondition for the proper functioning of a monetary economy, otherwise there will be sources and sinks, or sectors emitting money and sectors where money are accumulated without connection between the former and the latter. For example, the Central bank injects money via buying government bonds, but also withdraws it in the case of selling securities.

The flows on the Figure 1 are similar to the traditional Keynesian visualization of the circular flow of goods, services and payments in a closed economy.

We can imagine any economic system as a complex graph, where each vertex represents a particular sector, product, market, or economic agent involved in the production and exchange.

Edges between the nodes reflect the ties among economic entities. From a monetary point of view, these links are payments for goods, services or financial instruments. Since the economy is an interconnected system, any economic agent or market should in some way be linked to some other agent as a producer, receiving payments for the supplied goods and services and as a participant, paying for the obtained inputs.

In the graph theory, we have two basic approaches to the analysis of the connectivity of the system's elements. These are the cycles of Euler and Hamilton. In the case of Euler cycles we assume the existence of such a path within the graph that we can visit all *edges* without repetition, recurrences being admissible for the vertices only. In the Hamilton cycles case, the objective is to visit all the *vertices* without recurrence. Figure 1 is a clear example where both Euler and Hamilton cycles are realized. For example the cycle 12345 is a Hamiltonian one, while the cycle 524512345 is of the Euler type. The Hamiltonian cycle includes all vertices, but not all edges, while the Eulerian cycle covers all arrows and vertices.

From the point of view of monetary circulation both Hamiltonian and Euler cycles are of interest for us, as they guarantee the possibility of forming a closed circulation systems containing all economic agents and markets. One of the paradoxes of the graph theory is that the composition of Hamiltonian cycles is significantly more complicated than the construction of Eulerian ones (Wilson, 1996), that can be performed in linear time.

In particular, there is no general mathematical formalism for finding Hamilton cycles in complex graphs (Mertens, 2002), the problem being considered as NP-complete, what means that we need enormous computing time for calculating the possibilities for constructing Hamiltonian cycles with growing number of vertices (markets) and edges (cash payments between agents).

The neoclassical general equilibrium theory maybe associated with acyclic graphs since it is based on primitive matrices. If such matrices do not nave zero rows or columns, the computational complexity is relatively small (Blondel, Jungers and Olshevsky, 2015).

If we abandon barter exchange and introduce all functions of money, including means of exchange in particular, we increase the complexity, the markets are assumed to resolve. This is a result of the introduction of Euler cycles, necessary for describing intertwined money circulation cycles. Nevertheless the complexity of the problem is still limited to linear time.

The central problem of the economic theory is how to find set of prices that guarantee optimal allocation of resources. The neoclassical school is trying to resolve the problem in inconsistent and simplified way. If we introduce the price system consistently, as it is done in the second part of the present paper, the problem involves the configuration of Hamiltonian cycle and the task is already NP-complete.

We can go further and take into account the costs of arranging Hamiltonian cycle and assume expense minimization. This is necessary because in the real economies the prices are derived via affective circulation of money, what implies frictions and costs. In such a case the predicament is structured via the extensively studied traveling salesperson problem, which is seldom classified as NP-hard (Davendra, 2010). So the more we diverge from the neoclassical paradigm, the more complicated becomes the respective models.

The practical impossibility of artificial construction of complex Hamiltonian cycles drives the mathematical theory into another direction- linking graph theory with probability theory. This particularity of the graph theory is especially suitable for application in the field of finance and money, since the latter are necessarily related to concepts such as risk and uncertainty. Instead of explicitly construct or detect Hamilton cycles in complex graphs, we may simply postulate the existence of such objects (Hamilton random graphs) with a certain probability (Erdos and Renyi, 1960).

Studies show that closed Hamiltonian cycles are highly sensitive to the structure of relationships between elements. Under certain parameters of the system, closed loops may exist with a probability near unity (Brunet, 2005), while under other values of the parameters, the system breaks down into subgroups, distinguished by the presence of sources and sinks.

These features of the theory of random directed graphs allow for the modelling of processes such as phase transitions of the economic system from one mode to another. In particular, it we can model the transition from a command (which does not necessarily require the presence of closed loops) to a market economy or the shift from the phase of boom (expansion cycles) to bust (disruption of monetary circuits). Therefore, the crisis divides the economy into sectors accumulating financial assets (sources of funds) and sectors with increasing liabilities. At the beginning of the current financial crisis sources were the financial institutions investing in mortgage backed bonds and sinks represented liabilities, related to the accumulation of real estate.

Conducting monetary and fiscal policy (refinancing and recapitalization of commercial banks with the help the Treasury) is nothing but a creation of new connections in the payments chains in order to replace the broken links, that have led to an interruption of the system's connectivity and formation of sink-source breaks. Such policies allow to close the monetary circulation and to create new Eulerian and Hamiltonian cycles with high probability.

If we further develop this approach and view the configuration of cycles as random process of selecting new edges (bilateral monetary payment for goods, services and financial instruments), connecting vertices (economic agents), then the graphs of a size marginally less than a certain ceiling are unlikely to have a given property (Hamiltonian cycles), whereas graphs with slightly more edges are almost guaranteed to have it (this is referred to as phase transition, see Brunet, 2005).

In terms of complex money intermediated economic systems, this attribute can be qualified as the paradox of uncertainty, since the existence of random elements (uncertain money intermediated exchanges), is the basis of almost fail-safe systems of closed integrating cycles, while premeditated constructive efforts of selecting "infallible" configurations, are hopeless.

Overall, the use of the graph theory allows for the creation of adequate micro foundations of the monetary analysis and for relating the processes at macro level to the connectivity between participants in the money intermediated exchange.

5. Conclusions

The main conclusion of the present paper is that the appropriate inclusion of money and monetary circulation into economic analysis implies shift from acyclic to cyclic economic models. In the same time such transition allows for the analysis of more complex economic instances. It becomes possible to explain micro preferences by macro variables. Remarkably, the emergence and development of complex cyclical economic systems is actually possible only in the context of risk and uncertainty.

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